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QUESTIONS AND DISCUSSIONS.

Send all communications to U. G. Mitchell, University of Kansas, Lawrence.

NEW QUESTION.

34. Given the mixed integral and functional equation

$$\int_{x=0}^{x=x} f(x)dx = \frac{h}{6}\left[f(0) + 4f\left(\frac{x}{2}\right) + f(x)\right],$$

to determine the function $f(x)$. This equation is of rather fundamental practical value as it has to do with the most general solid whose volume is given by the prismatoid formula.

DISCUSSIONS.

I. CONCERNING AN ILLUSTRATION OF A CERTAIN NECESSARY CONDITION IN MINIMIZING A DEFINITE INTEGRAL WITH DISCONTINUOUS INTEGRAND.

By Paul R. Rider, Washington University.

The problem of minimizing a definite integral in which the integrand possesses a finite discontinuity along a plane curve has been considered by Bliss and Mason.[1] A certain necessary condition that they have discovered may be stated as follows:

If a curve C, of parameter t, which passes from the fixed point P_0 to the fixed point P_2 and crosses a curve D, whose equations are $x = x(\alpha)$, $y = y(\alpha)$, minimizes the sum of the two integrals

$$I = \int_{t_0}^{t_1} F(x, y, x', y')dt, \qquad i = \int_{t_1}^{t_2} f(x, y, x', y')dt,$$

the first integral to be taken from the point P_0 to the curve D and the second from the curve D to the point P_2, then at P_1, the point of intersection of C and D, the relation

$$x_\alpha(F_{x'} - f_{x'}) + y_\alpha(F_{y'} - f_{y'}) = 0$$

must hold. The argument of x_α, y_α is the value of α for the point P_1; the arguments of $F_{x'}$, $F_{y'}$ are the values of x, y, x', y' on the curve C_{01} at the point P_1, and those of $f_{x'}$, $f_{y'}$ are the values of the same variables on the curve C_{12} at the point P_1.

This condition is well illustrated in a problem (Calculus 389) which recently appeared in the Monthly:

> A man is at the southeast corner of a section of land and wishes to walk to the opposite corner in the least possible time. A circular track with a radius of $1/\pi$ miles is located in the section tangent to the west line at a point 120 rods from the south line. Conditions are such that he can walk at the rate of 4 miles an hour inside the track and 3 miles an hour outside the track. What course should he choose and how long is it? See the figure.

Solutions of the problem were given in the Monthly, Vol. 23, No. 24 (April, 1916), p. 125 by H. S. Uhler, and p. 127 by A. H. Holmes.

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[1] Bliss and Mason, "A Problem of the Calculus of Variations in which the Integrand is Discontinuous," *Transactions of the American Mathematical Society*, Vol. 7 (1906), pp. 325–336.

Since the time is to be the least possible, the sum of the three integrals

$$I_{01} = \int_{t_0}^{t_1} \frac{1}{3}\sqrt{x'^2 + y'^2}dt, \qquad i = \int_{t_1}^{t_2} \frac{1}{4}\sqrt{x'^2 + y'^2}dt, \qquad I_{23} = \int_{t_2}^{t_3} \frac{1}{3}\sqrt{x'^2 + y'^2}dt$$

must be minimized.

Here the curve D is the circle, the joint P_0 being the origin,

$$x = \frac{1}{\pi}\cos\alpha - 1 + \frac{1}{\pi}, \qquad y = \frac{1}{\pi}\sin\alpha + \tfrac{3}{8},$$

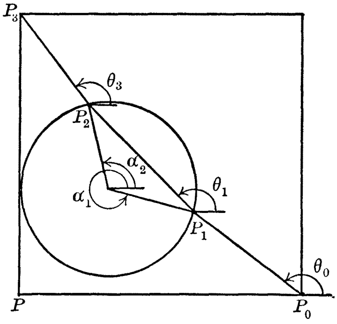
and the curve C is the broken line $P_0P_1P_2P_3$. (See figure.) There are two points of discontinuity, P_1 and P_2. For simplicity let us set

$$F = \tfrac{1}{3}\sqrt{x'^2 + y'^2}, \qquad f = \tfrac{1}{4}\sqrt{x'^2 + y'^2}.$$

It follows that the equations

$$(1) \qquad \begin{aligned} x_\alpha(F_{x'} - f_{x'}) + y_\alpha(F_{y'} - f_{y'}) &= 0, \\ x_\alpha(f_{x'} - F_{x'}) + y_\alpha(f_{y'} - F_{y'}) &= 0 \end{aligned}$$

must be satisfied at P_1 and P_2 respectively. But



$$x_\alpha = -\frac{1}{\pi}\sin\alpha, \qquad y_\alpha = \frac{1}{\pi}\cos\alpha,$$

$$(2) \quad F_{x'} = \frac{x'}{3\sqrt{x'^2 + y'^2}}, \qquad F_{y'} = \frac{y'}{3\sqrt{x'^2 + y'^2}}.$$

$$f_{x'} = \frac{x'}{4\sqrt{x'^2 + y'^2}}, \qquad f_{y'} = \frac{y'}{4\sqrt{x'^2 + y'^2}}.$$

If θ is the angle that a given line makes with the positive x-axis (taking PP_0 as the positive x-direction), then

$$F_{x'} = \tfrac{1}{3}\cos\theta, \qquad F_{y'} = \tfrac{1}{3}\sin\theta, \qquad f_{x'} = \tfrac{1}{4}\cos\theta, \qquad f_{y'} = \tfrac{1}{4}\sin\theta.$$

If we use these values, and those of x_α, y_α taken from (2), equations (1) assume the form

$$(3) \qquad \begin{aligned} -\sin\alpha_1(\tfrac{1}{3}\cos\theta_0 - \tfrac{1}{4}\cos\theta_1) + \cos\alpha_1(\tfrac{1}{3}\sin\theta_0 - \tfrac{1}{4}\sin\theta_1) &= 0, \\ -\sin\alpha_2(\tfrac{1}{4}\cos\theta_1 - \tfrac{1}{3}\cos\theta_2) + \cos\alpha_2(\tfrac{1}{4}\sin\theta_1 - \tfrac{1}{3}\sin\theta_2) &= 0. \end{aligned}$$

The meanings of α_1, α_2, θ_0, θ_1, θ_2 are evident from the accompanying figure.

From the solutions of the problem by Professor Uhler and Mr. Holmes are obtained the values

$$\begin{array}{ll} \alpha_1 = 360^\circ - 15^\circ\, 45'\, 25'', & \theta_0 = 90^\circ + 52^\circ\, 45'\, 48'', \\ \alpha_2 = 180^\circ - 75^\circ\ \ 6'\, 19'', & \theta_1 = 90^\circ + 44^\circ\ \ 5'\, 13'', \\ & \theta_2 = 90^\circ + 37^\circ\ \ 1'\, 51''. \end{array}$$

Whence,

$$\begin{array}{lllll} & \sin\alpha_1 = -\,.271557, & \cos\alpha_1 = .962422, & \sin\theta_0 = .605109, & \cos\theta_0 = -\,.796143, \\ (4) & \sin\alpha_2 = .966400, & \cos\alpha_2 = -\,.257044, & \sin\theta_1 = .718285, & \cos\theta_1 = -\,.695749, \\ & & & \sin\theta_2 = .798312, & \cos\theta_2 = -\,.602245. \end{array}$$

If we substitute these numerical values in the first equation of (3) the left side becomes

$$.271557(-\tfrac{1}{3} \times .796143 + \tfrac{1}{4} \times .695749) + .962422(\tfrac{1}{3} \times .605109 - \tfrac{1}{4} \times .718285) = .003532.$$

Substituting the values (4) in the second equation of (3), we get for its left side

$$-\,.966400(-\tfrac{1}{4} \times .695750 + \tfrac{1}{3} \times .602245) - .257044(\tfrac{1}{4} \times .718285 - \tfrac{1}{3} \times .798312) = -\,.003668.$$

Thus it is seen that the values (4) approximately satisfy conditions (1).

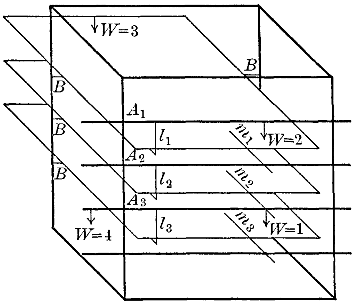
It is to be noted that in Professor Uhler's solution use is made of the law of refraction. Since this law is a special case of the Bliss-Mason condition, it follows that if Professor Uhler's results are used, the foregoing serves as a check on his solution.

II. Relating to an Equation Balance.

By E. L. Rees, University of Kentucky.

The equation balance described below, though less simple in design and construction than some others, offers, in the opinion of the writer, many compensating advantages.

This machine consists of a frame supporting two sets of levers. The levers of one set, which we shall call the scale levers, have their fulcrums at the points marked A in the figure. Just back of, and on a level with, each scale lever, except the top one, is a platform lever supported at its center on the knife edges marked B. These two sets of levers are connected in the following manner: A point one unit from the fulcrum on the first scale lever is joined to the first platform lever by a connecting rod l_1; similarly the second scale lever is connected to the second platform lever, etc. On each platform lever is a movable arm (m_1, m_2, m_3) resting on the corresponding scale lever. This arm passes through a slot in the scale lever so that all vertical forces may be transmitted through the arm to the scale lever.



The following example will serve to illustrate both the principle and the operation of the machine.

Let us solve the cubic equation $2x^3 - 3x^2 + x - 4 = 0$. Having balanced all the levers, we place on the first scale lever a weight 2, representing the first coefficient, in the equation. We then place the movable arms (m_1, m_2, m_3) in the same vertical line with this weight. Calling